CHIZHKOV, B., tokar'; VERGEYCHIK, A., tokar'; SMIRNOV, M.; KRASOVSKIY, N.; SHITYKO, P.; CHAYKA, D.; MAZURENKO, P.

Same conditions bring different results. Okhr. truda i sots. strakh. no.1:30-33 Jl '58. (MIRA 11:12)

1. Instrumental'nyy tsekh Minskogo podshipnikovogo zavoda (for Chizhkov, Vergeychik). 2. Starshiy inzhener po tekhnike bezopasnosti Minskogo podshipnikovogo zavoda (for Smirnov). 3. Sekretar' redaktsii zavodskoy mnogotirazhki "Za tekhnicheskiy progress" Minskogo podshipnokovogo zavoda (for Krasovskiy). 4. Glavnyy tekhnicheskiy inspektor Belsovprofa (for Shityko). 5. Spetsial'nyy korrespondent zhuranla Vsesoyuznogo tsentral'nogo soveta profsoyuzov "Okhrana truda i sotsial'noye strakhovnaiye" (for Mazurenko).
(Minsk--Industrial hygiene)

SOV/111-58-12-16/36

AUTHORS: Garash, B.S., Chief, Krasovskiy, N.G., Senior Engineer

TITLE: The Introduction and Operation of UPTS Equipment in the Intra-Rayon Telephone Network (Vnedreniye i ekspluatatsiya ustroystv UPTS na seti vnutrirayonnoy telefonnoy svyazi)

PERIODICAL: Vestnik svyazi, 1958, Nr 12, p 15 (USSR)

ABSTRACT: The authors review the experiences of the communication workers of the Moldavian SSR in introducing and operating UPTS equipment, which began in 1956. Many difficulties had to be overcome which were caused by differences in the various existing telephone networks and the manual and automatic telephone offices to which the semiautomatic telephone office equipment (UPTS) had to be connected. There is 1 circuit diagram.

ASSOCIATION: Direktsiya radiotranslyatsionnoy seti i vnutrirayonnoy telefonnoy svyazi Moldavskoy SSR (Central Office of Radio Relay and Rayon Telephone Network of the Moldavian SSR).

Card 1/1

KRASOVSKIY, N. N.

Ordinary Differential Equations

Dissertation: "Stability of Motion for Any Initial Disturbances." Cand Phys-Math Sci, Ural Polytechnic Inst, Sverdlovsk, 1953. (Referativnyy Zhurnal -- Matematika Moscow, Mar 54).

SO: SUM 213, 20 Sep 1954

KRASOVSKIY, N. N.

USSR/Mathematics - Stability

Card 1/1

Author : Krasovskiy, N. M.

Title : Stability of motion in the large for constantly acting disturbances

Periodical : Prikl. mat. i mekh., 18, 95-102, Jan/Feb 1954

Abstract : Examines five separate systems of differential equations. Discusses for each the consequences of asymptotic stability of the solutions as well as stability in the large for constantly acting disturbances. Proves four important theorems relating stability and uniqueness to initial conditions imposed upon the systems.

Institution : Ural Polytechnic Institute, Sverdlovsk

Submitted : April 8, 1953

GRIB, V.K.; KRASOVSKIY, N., red.; BABENKOVA, A., spets. red.

[Some problems of the mechanization of loading and unloading operations on fish farms] Nekotorye voprosy mekhanizatsii pogruzochno-razgruzochnykh rabot v prudovykh khoziaistvakh. Minsk. Belorusskoe pravlenie NTO pishchevoi promyshl. 1962. 24 p. (MIRA 17:3)

1. KRASOVSKIY, N.N.

2. USSR (600)

4. Motion

7. Theorems on stability of motions, determined by a system of two equations. Prikl. mat. i mekh. 16, no. 5, 1952.

9. Monthly List of Russian Accessions, Library of Congress, February 1953. Unclassified.

KRASOVSKIY, N.N.

Barbalin, E. A., and Krasovskii, N. N. On stability of motion in the large. Doklady Akad. Nauk SSSR (N.S.) 86, 453–456 (1952). (Russian)

The authors consider differential equations (1) $\dot{x}=X(x)$, where x is a vector in n-dimensional space and the components X_i of X are of class C' everywhere, with $X(0)=0$. The solution $x=0$ is said to be asymptotically stable for arbitrary initial conditions if (*) every solution $x(t)$ of (1) tends to 0 as $t\to+\infty$. Existence of a differentiable function $v(x)$, everywhere positive, such that $\dot{v}<0$ on each solution would, according to Liapounoff's theory, guarantee a local stability of the solution $x=0$. It is shown by an example that this need not imply (*). If, in addition, $v(x)\to\infty$ as $\|x\|\to\infty$, then it is shown that (*) does follow. Conversely, if (*) holds, then such a v does exist. If the condition $\dot{v}<0$ is replaced by the conditions: $\dot{v}<0$ outside of M, $\dot{v}\leq 0$ in M, and M is a set such that the intersection of each level surface $v=c$ and M contains no positive semi-trajectory of (1), then (*) again follows. It is shown that these hypotheses are satisfied by the system of second order: $\dot{x}=y$, $\dot{y}=-\phi(y)-g(y)f(x)$, provided $f(0)=\phi(0)=0$, $xf(x)>0$ for $x\neq 0$, $y\phi(y)>0$ for $y\neq 0$, $g(y)>0$, $\int_0^x f(x)dx\to\infty$ as $|x|\to\infty$ and $\int_0^y yg(y)^{-1}dy\to\infty$ as $y\to\infty$. In the proofs use is made of results of a previous paper by Barbašin [Mat. Sbornik N.S. 29(71), 233–280 (1951); these Rev. 13, 756].

W. Kaplan (Ann Arbor, Mich.).

KRASOVSKIY, N. N.

"Stability for Any Initial Disturbances of the Solutions of a Certain Nonlinear System of Three Equations," Prik. Mat. i Mekh., 17, No.3, pp 339-350, 1953.

Ural Polytech. Inst., Sverdlovsk.

Demonstrates the sufficient conditions to be imposed on the functions $f_i(x)$ ($i=1,2,3$) in order that the solution $x=y=z=0$ of the following system be asymptotically stable for any initial perturbations: $dx/dt=f_1(x)+a_1y+b_1z$, $dy/dt=f_2(x)+a_2y+b_2z$, $dz/dt=f_3(x)+a_3y+b_3z$, where a_1, b_1 are constants and functions $f_i(x)$ ($f_i(0)=0$) are continuous. Cites related works of N.P. Yerugin ("Certain Problem of the Theory of Stability of Automatic Regulation Systems," ibid. Vol. 16, No.5, 1952) and others.

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KRASOVSKIY, N.N. (Sverdlovsk)

Stability of the solution of a system of two differential equations. Prikl.mat.i mekh. 17 no.6:651-672 N-D '53. (MLRA 6:12)

1. Ural'skiy politekhnicheskiy institut.
(Differential equations)

KRASOVSKIY, N.N.

GANAGO, O.A., kandidat tekhnicheskikh nauk; TARNOVSKIY I.Ya., professor, doktor tekhnicheskikh nauk; KRASOVSKIY, N.N., inzhener.

Designing optimum blank shapes for forging gear-type products. Trudy Ural.politekh.inst. no.45:137-151 '53. (MLRA 9:11)
(Forging)

1. KRASOVSKIY, N.N.

2. USSR (600)

4. Stability

7. Problem of the stability of motion in the large, Dokl. AN SSSR, 88, No. 3, 1953.

9. Monthly List of Russian Accessions, Library of Congress, April, 1953, Uncl.

APPROVED FOR RELEASE: Monday, July 31, 2000 CIA-RDP86-00513R00082621OC

KRASOVSKIY, N.N.

Krasovskii, N. N. On stability of solutions of a system of second order in critical cases. Doklady Akad. Nauk SSSR (N.S.) 93, 965–967 (1953). (Russian) 62

A stability criterion is established for a borderline case for the equations $\dot{x}_i = X_i(x_1, x_2)$ $(i = 1, 2)$. It is assumed that the X_i vanish at $(0, 0)$ and have partial derivatives satisfying a Lipschitz condition in a neighborhood of $(0, 0)$. Let $p(\lambda; x_1, x_2) = \det(\|\partial X_i/\partial x_j\| - \lambda I)$, where I is the unit matrix and let $p(\lambda; 0, 0)$ have two imaginary roots or one zero root and one negative root. It is then proved that if, for each (x_1, x_2) other than $(0, 0)$, $p(\lambda; x_1, x_2)$ has roots with negative real parts, then $(0, 0)$ is asymptotically stable, whereas, if, at each (x_1, x_2), $p(\lambda, x_1, x_2)$ has roots with positive real parts, then $(0, 0)$ is unstable. *W. Kaplan.*

TARNOVSKIY, I.Ya., prof.; POZDEYEV, A.A., inzh.; KRASOVSKIY, N.N., inzh.

Force determination in metalworking by pressure. Obr.met.davl. no.3:5-22 '54. (MIRA 12:10)

1. Ural'skiy politekhnicheskiy institut im. S.M.Kirova.
(Rolling (Metalwork)) (Forging)

KRASOVSKIY, N.N.

Krasovskii, N. N. On stability of motion in the large for constantly acting disturbances. Akad. Nauk SSSR. Prikl. Mat. Meh. 18, 95–102 (1954). (Russian)

Consider the n-vector equation

(1) $\dot{x}=X(x), \quad X(0)=0$

where X is continuous and the associated system

(2) $\dot{x}=X(x)+R(x;t),$

where generally $R(0;t)\neq 0$. The origin is said to be stable in the large for constantly acting disturbances (=s.l.c.d.) if it is stable in the small for constantly acting disturbances [see the book reviewed second above] and furthermore the following holds: Let $\|x\|=\sup|x_i|$, and similarly for other vectors. Then, given ϵ, there exists η such that if R is such that outside $\|x\|<\epsilon$, $\|R\|<\eta f(r)$, $r=(\sum x_i^2)^{1/2}$, then every solution of (2) has the property that lim sup $\|x(t)\|\to 0$ as $t\to+\infty$. Here $f(r)$ is a given function characterizing the admissible growth of R.

The object of the present paper is to give sufficient condition for s.l.c.d. for certain systems considered in various papers [Erugin, these Rev. 14, 376; Krasovskiĭ, ibid. 14, 376; Malkin, ibid. 14, 48; Eršov, ibid. 14, 752]. The assumption is made throughout that $f(r)=r$, and x, y are now the plane cartesian coordinates. Consider the system (over)

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(3) $\dot{x} = F(x, y), \quad \dot{y} = f(s), \quad s = ax - by,$

where a, b are constants with $a \neq 0$, F and f are continuous, $F(0,0) = f(0) = 0$, and F, f satisfy the condition for uniqueness of solutions at the origin. Then: Theorem. If the solutions of the equation

$$\begin{vmatrix} F_x - \lambda & F_y \\ f_x & f_y - \lambda \end{vmatrix} = 0$$

have their real parts $< -\delta$, where δ is a suitably small positive number, and this for all x, y, then the system (3) is s.l.c.d.

Consider now the system

(4) $\dot{x} = f_1(x) + ay, \quad \dot{y} = f_2(x) + by,$

where a, b are constants with $a \neq 0$, the f_i are continuous with $f_i(0) = 0$, and $|f_i(x)| < M|x|$, where M is a suitably large constant. Then the same theorem holds relative to the roots of (CONT)

KRASOVSKII, N. N.

$$\begin{vmatrix} x^{-1}f_1(x)-\lambda & a \\ x^{-1}f_1(x) & b-\lambda \end{vmatrix} = 0.$$

Finally, the same theorem holds regarding the system

(5) $\dot{x}=f_1(x)+ay, \quad \dot{y}=bx+f_2(y), \quad |f_i(s)|<M|s|,$

as regards the roots of

$$\begin{vmatrix} x^{-1}f_1(x)-\lambda & a \\ b & y^{-1}f_2(y)-\lambda \end{vmatrix} = 0.$$

[Additional references: Barbašin and Krasovskiĭ, these Rev. 14, 646; Barbašin, ibid. 14, 376; Krasovskiĭ, ibid. 14, 1087.]

S. Lefschetz (Princeton, N. J.).

LB

6-30-55

KRASOVSKIY, N.N.

Krasovskiĭ, N. N. On the behavior in the large of the integral curves of a system of two differential equations. Akad. Nauk SSSR. Prikl. Mat. Meh. 18, 149–154 (1954). (Russian)

Let $\dot{x}=X(x,y)$, $\dot{y}=Y(x,y)$, where X, Y have continuous first partials for any (x,y) (certain weaker assumptions are also considered), $X(0,0)=Y(0,0)=0$; let J be the Jacobian matrix of X, Y at any point (x,y), $\lambda_i(x,y)$, $i=1,2$, its characteristic roots, $m(r)=\inf(X^2+Y^2)^{1/2}$, $M(r)=\sup(X^2+Y^2)^{1/2}$ for $x^2+y^2=r^2$. The author proves: (i) if $\operatorname{Re}\lambda_i<0$ for $x^2+y^2\leq R_1^2$ and $\int_R^{R_1} m(r)dr>2\pi RM(R)$, then the circle $x^2+y^2\leq R^2$ belongs to the attractive domain of $(0,0)$; (ii) if $\operatorname{Re}\lambda_i<0$ in the whole plane and $\int_0^\infty m(r)dr=\infty$, the origin is stable in the large; (iii) if the origin is the only singular point, $\operatorname{Re}\lambda_i(0,0)>0$, $\operatorname{Re}\lambda_i(x,y)<0$ for $x^2+y^2=R^2$ and $\int_0^\infty m(r)dr=\infty$, there exists at least one stable limit cycle.

J. L. Massera (Montevideo).

[illegible]IY, [illegible]

USSR/Mathematics - Asymptotic stability

FD-640

Card 1/1 : Pub. 85 - 7/12

Author : Barbshin, Ye. A., and Krasovskiy, N. N. ([illegible]lovsk)

Title : Existence of the Lyapunov functions in the case of asymptotic stability in the whole

Periodical : Prikl. mat. i mekh., 18, 345-350, May/Jun 1954

Abstract : Treats the system of equations $dx/dt = X(x,t)$. Demonstrates that even in the case where the right part of this equation depends upon the problem of the existence of the Lyapunov function is solved in the positive sense for sufficiently general assumptions on the system. His formulation is similar to that of I. G. Malkin ("Problem of the conversion of A. M. Lyapunov's theorem on asymptotic stability," PMM, 18, No. 2, 1954). Five references, including I. Massera, Liapounoff's condition of stability, Annals of Mathematics, No. 50, No. 3, 1949.

Institution : --

Submitted : March 16, 1954

KRASOVSKIY, N. N.

USSR/Mathematics - Theorem of Lyapunov

FD-947

Card 1/1 Pub 85-1/11

Author : Krasovskiy, N. N.

Title : Reciprocity of theorems of A. M. Lyapunov and N. G. Chetayev on instability for stationary systems of differential equations

Periodical : Prikl. mat. i mekh. 18, 513-532, Sep/Oct 1954

Abstract : I. L. Massera (On Liapunoff's condition of stability, Annals of Mathematics, Vol 50, No 3 (1949)) and Ye. A. Barbashin, (Method of cross sections in the theory of dynamic systems, Mat. sb. Vol 29, No 2 (1951)) proved that Lyapunov's theorem on asymptotic stability may be reciprocal. This problem is further analyzed and solved by finding a function $v(x_1,\ldots,x_n)$ satisfying the conditions in the instability theorems of Lyapunov (Obshchaya zadacha ob ustoychivosti dvizheniya (General problem on stability of motion) 1950) and of Chetayev (Ustoychivost dvizheniya (Stability of motion) 1946). Indebted to N. G. Chetayev. Eight references including one aforementioned US.

Institution : --

Submitted : April 9, 1954

KRASOVSKIY, N.N.

USSR

Krasovskiĭ, N. N. On stability in the large of the solution of a nonlinear system of differential equations. Prikl. Mat. Meh. 18, 735–737 (1954). (Russian)

Let the n-vector system (1) $\dot{x}=X(x)$ be such that $X(0)=0$ and that X is of class C^1 in the whole space. Let J denote the Jacobian matrix of X. Theorem: In order that the origin be asymptotically stable for the whole space it is sufficient that there exist a symmetric matrix A such that for all x the characteristic roots of $\frac{1}{2}(AJ+J'A)$ all lie below a certain fixed $-\delta$, $\delta>0$. [Additional reference: Erugin, Prikl. Mat. Meh. 15, 227–236 (1951); MR 12, 705.] *S. Lefschetz.*

KRASOVSKIY, N.N.

USSR/Physics - Metallurgy

Card 1/1 Pub. 147 - 20/27

Authors : Krasovskiy, N. N.; Nikitin, Yu. P.; Esin, O. A.; and Popel', S. I.

Title : Calculation of surface tension by the form of a recumbent drop

Periodical : Zhur. fiz. khim. 28/9, 1678-1679, Sep 1954

Abstract : A table for the calculation of surface tension according to the form of a recumbent drop and a suitable method for the graphical integration of an equation for such a drop are briefly described. The method, which has numerous advantages, is also applicable to drops of different size and form. Examples of such calculations are shown. Five references: 3-USSR; 1-Indian and 1-English (1883-1953). Table; graph.

Institution : The S.M.Kirov Ural Polytechnicum, Faculty of the Theory of Metallurgical Processes, Sverdlovsk

Submitted : April 20, 1954

KRASOVSKIY, N.N

USSR/Mathematics

Card 1/1 : Pub. 22 - 5/44

Authors : Krasovskiy, N. N.

Title : Sufficient conditions for the stability of solutions of a system of non-linear differential equations

Periodical : Dok. AN SSSR 98/6, 901-904, October 21, 1954

Abstract : Sufficient conditions for the stability of solutions of a system of non-linear differential equations, such as

$$\frac{dx_i}{dt} = X_i(x_1, x_2, \ldots, x_n) \quad (i = 1,2,\ldots,n)$$

are sought. The results of this search is presented in three theorems. Four references (1950-1953).

Institution : Ural'skiy Polytechnical Institute im. S. M. Kirov

Presented by: Academician I. G. Petrovskiy, June 10, 1954

KRASOVSKIY, N.N.

Krasovskii, N. N. On inversion of K. P. Persidskii's theorem on uniform stability. Prikl. Mat. Meh. 19, 273–278 (1955). (Russian) 1-F/W

62

This note deals with the uniform stability of the origin for a system

$$\dot{x} = X(x; t), \quad X(0; t) = 0, \quad \|x\| \leq H, \quad t > 0,$$

where x, X are n-vectors and the modulus is sup $\|x_i\|$. The uniform stability is in the sense of Persidskii (with respect to the time). In substance the author proves: A necessary and sufficient condition for the uniform stability of the origin is the existence of a Lyapunov function $v(x; t)$ which is definite positive, is small uniformly in t ($t \geq t_0 > 0$), and has a time derivative $\dot{v}$ (along the trajectories) which is negative. The necessity of the condition was already proved by Persidskii [Dissertation, Moscow, 1946].

S. Lefschetz (Princeton, N. J.).

KRASOVSKIY, N.N.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/3 PG - 45
AUTHOR KRASOVSKIJ N.N.
TITLE On the stability after the first approximation.
PERIODICAL Priklad. Mat. Mech. 19, 516-530 (1955)
reviewed 5/1956

The author considers the system

(1) $$\frac{dx_i}{dt} = X_i(x_1,x_2,\dots x_n,t) \qquad i=1,\dots,n\ ,$$

where the functions X_i are defined in the range $\mathcal{L}(|x_i| \le H,\ 0 \le t)$ and are continuous in t and x_i and there possess continuous derivatives $\frac{\partial X_i}{\partial x_j}$ which satisfy the inequations $\left|\frac{\partial X_i}{\partial x_j}\right| < \mathcal{L}$. H and $\mathcal{L}$ are constants. $X_i(0,0,\dots 0,t) = 0$ for $t \geq 0$. As a positive (or negative) halftrajectory the maximum connecting bow of a trajectory is denoted which lies in the range $\mathcal{L}$ for $t \geq t_0$ (or $t \leq t_0$). If it is assumed that the solutions $x_i(x_{10},\dots,x_{n0},t_0,t)$ satisfy the following condition: (2) there exist such constants $\alpha > 0$, $\beta > 0$, that for every point p of $\mathcal{L}$ on at least one half-trajectory the inequation is satisfied by p:

Priklad. Mat. Mech. 19, 516-530 (1955)

$$\ln r(p,t_o,t) \geqslant \ln \beta r(p) + \alpha|t - t_o| \quad (\text{for } t \geqslant t_o \text{ or } \leq t_o).$$

Here mean: $r(p) = (x_{1p}^2 + \ldots + x_{np}^2)^{1/2}$ and

$$r(p,t_o,t) = \left[x_1^2(x_{1p},\ldots x_{np},t_o,t)+\ldots+ x_n^2(x_{1p},\ldots,x_{np},t_o,t_1)\right]^{1/2} .$$

It is proved that the condition (2) is necessary and sufficient for the existence of a function $v = v(x_1,x_2,\ldots x_n,t)$ which satisfies the inequations

(3) $$|v| \leqslant c_1 r^A, \ \frac{dv}{dt} \geqslant c_2 r^A, \ \left|\frac{\partial v}{\partial x_i}\right| < c_3 r^{A-1},$$

where A, c_1, c_2, $c_3 > 0$ are constant and $r = (x_1^2 + \ldots + x_n^2)^{1/2}$. This function v then also satisfies the Liapunov conditions, and the asymptotic stability (or instability) of the movement is determined by the behavior of the linear approximation and does not depend on terms of higher order.
For the system

$$\frac{dx_i}{dt} = \sum_{j_1=1;\ldots;j_n=1}^{j_1+\ldots+j_n=m} P_{ij_1\ldots j_n}(t)\, x_1^{j_1}\ldots x_n^{j_n} \qquad (i=1,2,\ldots n; m \quad 1)$$

a result of Malkin (Doklady Akad. Nauk 76, No.6 (1951)) is generalized. The variation of the attitude of the solution is investigated for additional terms of order higher than m, and it is stated that, under certain conditions, Liapunov functions are existing which satisfy certain estimations. Then the stable or instable attitude of the solutions is not disturbed by additional terms of order higher than m, if the absolute values of these terms lie below a definite limit:

$$\left|R_i(x_1,\ldots x_n,t)\right| < a(|x_1| + \ldots + |x_n|)^m.$$

Exemples are given for the application of the obtained results to systems with delays of t.

KRASOVSKIY, N.N.

Krasovskii, N. N. On stability of motion in the critical case of a single zero root. Mat. Sb. N.S. 37(79) (1955), 83–88. (Russian) 1-F/W

Consider a system (1) $\dot{x}=X(x)$, where x, X are $(n+1)$-vectors, $X(0)=0$ and X is of class C' at the origin. Suppose also that the matrix A of the first-degree terms has one characteristic root zero and the rest with negative real parts. Let $J=\|\partial X_i/\partial x_j\|$. Theorem: If in a neighborhood of the origin (origin excepted) the characteristic roots of J have their real parts all negative [some positive] then the origin is asymptotically stable [is unstable] for (1).

This question was already dealt with by Lyapunov [Problème général de la stabilité du mouvement, Princeton, 1947; MR 9, 34] who first reduced (1) to a special form. His condition reads decidedly differently. The present author's starting point is Lyapunov's reduced system. His condition offers the decided advantage that it is expressed directly in terms of (1) without requiring a preliminary transformation. [Additional references: Krasovskii, Dokl. Akad. Nauk SSSR (N.S.) 93 (1953), 965–967; MR 15, 795; Četaev, Stability of motion, Gostehizdat, Moscow, 1946].

S. Lefschetz (Princeton, N.J.).

KRASOVSKIY, N.N.

USSR/Mathematics

Card 1/1 Pub. 22 - 4/51

Authors : Krasovskiy, N. N.

Title : About the conditions for convergence of Lyapunov's theorems on the instability of stable systems of differential equations

Periodical : Dok. AN SSSR 101/1, 17-20, Mar. 1, 1955

Abstract : Conditions under which the functions satisfying Lyapunov's theorems on instablity may exist are discussed. Conditions of the so-called "assymptotic instability" are also considered. Four references: 3 USSR and 1 English (1941-1951).

Institution : The S. Kirov Ural Polytechnicum

Presented by : Academician A. N. Kolmogorov, December 14, 1954

KRASOVSKIY, N.N.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/1 PG - 540
AUTHOR KRASOVSKIJ N.N.:
TITLE On the question of the reversal of the theorems of the second Liapunov method for the investigation of the stability of motion.
PERIODICAL Uspechi mat.Nauk 11, 3, 159-164 (1956)
reviewed 1/1957

A new quite simple method of proof of the following theorems is given: 1. Theorem of Barbašin on the existence of a section in a dispersive dynamical system (Mat.Sbornik, n. Ser. 29, 233-280 (1951)); 2. Theorem of Barbašin (ibid.) and Massera (Annals of Math. 50, 705-721 (1949)) on the inversion of Liapunov's theorem on asymptotic stability for autonomous systems; 3. The following theorem on the inversion of Liapunov's theorem on instability: Let $\dot{x} = X(x,t)$ where X is continuously differentiable, $X(0,t) = 0$ be such that $x = 0$ is unstable; then there is a continuously differentiable function $v(x,t)$ having an infinitely small upper bound the total derivative of which $\frac{dv}{dt} = \lambda v + w$, $\lambda > 0$, $w \geqslant 0$, and such that $v > 0$ for any $t = t_0 > 0$ and x arbitrarily near $x = 0$.

KRASOVSKIY, N.N.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/1 PG - 382
AUTHOR KRASOVSKIJ N.N.
TITLE The reversal of the theorems of Liapunov's second method and the problem of the stability of motion after the first approximation.
PERIODICAL Priklad. Mat. Mech. 20, 255-265 (1956)
reviewed 11/1956

The present paper essentially represents a combination and coordination of several results of the author and of such ones of other authors which refer to the following themes: 1) Relation between the existence of the Liapunov function and the notion of uniform stability. 2) Characteristic of a method (N.N.Krasovskij, Priklad. Mat. Mech. 18, 5, (1954) and 19, 5, (1955)) for the reversal of the theorems of Liapunov's second method which is applicable in the case of stability as well as in the case of instability. 3) Proof of existence for Liapunov's functions which satisfy certain estimations and application to the investigation of stability problems after the first approximation.
In the last section the author formulates with sketched proofs some new results concerning the application of Liapunov's functions for systems with delay (the paths of motion are considered in the abstract space).

INSTITUTION: Sverdlovsk.

KRASOVSKIY, N.N.

3

Krasovskii, N. N. On the application of the second method of Lyapunov for equations with time retardations. Prikl. Mat. Meh. 20 (1956), 315-327. (Russian)

Math

Lyapunov's second method is extended to systems

$$\dot{x}_i(t)=X_i[x_1(t-h_{i1}(t)), \cdots, x_n(t-h_{in}(t)), t], \quad 1\leq i\leq n,$$

where $X(x, t)$ is lipschitzian on $\|x\|\leq H$, $t\geq 0$ and $X(0, t)\equiv 0$ for $t\geq 0$, and where the functions $h_{ij}(t)$, $1\leq i, j\leq n$, are piecewise continuous and satisfy $0\leq h_{ij}(t)\leq h_j=\text{const}$ for $t\geq 0$. After suitable modification of the basic stability definitions, the author proves sufficient conditions for the asymptotic and total stability of $x=0$, necessary and tion. After suitable modification of the basic stability definitions, the author proves sufficient conditions for the asymptotic and total stability of $x=0$, necessary and sufficient conditions for the uniform asymptotic stability and theorems relating exponential asymptotic stability to total stability. Their precise statements are too long to be stated more explicitly here. All results are based upon considerations of positive definite functions in the manner of Lyapunov's second method. *H. A. Antosiewicz*

KRASOVSKIY, N.N.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/3 PG - 173
AUTHOR KRASOVSKI N.N.
TITLE On the asymptotic stability of the systems with retardation.
PERIODICAL Priklad. Mat. Mech. 20, 513-518 (1956)
reviewed 11/1956

The author considers the motion equations

$$(1) \qquad \frac{dx_i}{dt} = X_i(x_1(t-\vartheta), \ldots, x_n(t-\vartheta), t) \qquad (i = 1, \ldots, n)$$

in which $X_i(y_1(\vartheta), \ldots, y_n(\vartheta))$ are functionals which for $t \geq 0$ are defined on piecewise continuous functions $y_j(\vartheta)$. Here $0 \leq \vartheta \leq h$, h positive constant; $\|y(\vartheta)\| < H$, H = const, also $H = \infty$; $X_i(0, 0, \ldots, 0, t) = 0$ for $t \geq 0$; $|X_i(y(\vartheta), t) - X_i(y^*(\vartheta), t)| < L \|y(\vartheta) - y^*(\vartheta)\|$. Under these conditions to every piecewise continuous curve $\|x(t_0 - \vartheta)\| < H$ for $t \geq t_0$ there corresponds a single solution of (1). This solution exists for all $t \geq t_0$ for which they are continuable in the region $\|y(\vartheta)\| < H$.
As initial curves $x_0(t_0 - \vartheta_0)$, which determine the motion paths, all piecewise continuous curves are allowed which satisfy the inequation

$$(2) \qquad \|x_0(t_0 - \vartheta_0)\| < G \qquad G < H.$$

The trivial solution $x = 0$ is called stable if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that from $\|x_0(t_0 \cdot \vartheta_0)\| < \delta$ there follows the inequation $\| x(x_0(t_0 \cdot \vartheta_0), t \cdot \vartheta)\|_h < \varepsilon$ for $t \geqslant t_0$. If for all piecewise continuous initial curves of (2): $x(t) \rightarrow 0$ as $t \rightarrow \infty$, then $x = 0$ is called asymptotically stable.

The following criterion of stability is proved. The trivial solution $x = 0$ is asymptotically stable if there exists a functional $V(y(\cdot \tau), t)$ which is defined on the functions $y(\cdot \tau)$ $(0 \leqslant \tau \leqslant h_1, h_1 \leqslant h)$ and which satisfies the following conditions:

1. V is positive definite with respect to a metric which is defined by the norm $\| y(\cdot \tau)\| = \sup | y_i(\cdot \tau)|$ $(0 \leqslant \tau \leqslant h_1)$.
2. V has an infinitely small upper bound on the curves $y(\cdot \tau)$, $\|y(\cdot \tau)\| < H$
3. V satisfies the inequation

$$\inf (V(y(\cdot \tau), t))_{G < G_1 \leqslant \| y \| \leqslant H_1 < H} > \sup(V(y(\cdot \tau), t))_{\| y \| \leqslant G}$$

4. $\overline{\lim}\, (\Delta V/\Delta t)$ $(\Delta t \rightarrow +0)$ is a negative definite functional on the continuous curves $y(t \cdot \vartheta)$ which, for $-h+t \leqslant \xi \leqslant t$ satisfy the inequation

$$V(y(\xi \cdot \tau), \xi) < f(V(y \cdot \tau), t);$$

here $f(r)$ is a continuous monotone function, $f(r) > r$ for $r \neq 0$.
Some examples explain the use of the criterion.

INSTITUTION: Moscow.

SUBJECT	USSR/MATHEMATICS/Differential equations	CARD 1/1	PG - 739
AUTHOR	KRASOVSKI N.N.		
TITLE	On the second Ljapunov method for investigations of stability.		
PERIODICAL	Mat.Sbornik,n.Ser. 40, 57-64 (1956) reviewed 5/1957		

The author gives necessary and sufficient conditions that there exists a Ljapunov function v which has a positive definite derivative and an infinitely small upper bound. The investigation relates to the non-stationary system

$$\frac{dx_i}{dt} = X_i(x_1, x_2, \dots, x_n, t) \qquad i=1,2,\dots,n.$$

Furthermore conditions for the reversibility of Ljapunov's theorem of instability are given. In the case of stationary systems one obtains earlier results of the author (Priklad.Mat.Mech. 18, 513-532 (1954)).

INSTITUTION: Sverdlovsk.

KRASOVSKIY, N.N.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/3 PG - 350

AUTHOR KRASOVSKIJ N.N.

TITLE On the theory of the second Liapunov method for the investigation of the motion stability.

PERIODICAL Doklady Akad. Nauk 109, 400-463 (1956)
reviewed 10/1956

The author considers the stability of the trivial solution $x_1 = x_2 = \dots = x_n = 0$ of the system

$$\frac{dx_i}{dt} = X_i(x_1, x_2, \dots, x_n, t) \qquad i=1,2,\dots n,$$

where the functions X_i in the region $|x_i| < H$, $t \geq 0$ are assumed to be continuously differentiable and satisfy the condition $\left|\frac{\partial X_i}{\partial x_j}\right| < L$ = const. Several definitions are given in order to establish the notions of the simple, the uniform and the asymptotic stability. A result of the author (Priklad.Mat.Mech. 19, 2, (1955)) is improved by the formulation of the following theorem without proof: For the asymptotic stability of the trivial solution it is necessary and sufficient that there exists a positive definite function $v(x_1, \dots x_n, t)$ of the class C_1 the derivative of which $\frac{dv}{dt}$ is negative semidefinite and satisfies the condition

Doklady Akad. Nauk 109, 400-463 (1956)

$$\int_0^\infty m_\eta(\tau)d\tau = -\infty \quad \text{for all sufficiently small } \eta > 0.$$

Here $m_\eta(\tau)$ is a continuous function such that in the region $\eta \leq v(x_1,\ldots,x_n,\tau)$, $|x_i| \leq \sigma$, $\tau = t$ the inequation

$$m_\eta(\tau) \geqslant \sup \frac{dv}{dt}$$

is satisfied.
Then it is shown that the method of Liapunov functions can be applied to investigations of stability with respect to first approximation in a metric space R. The obtained generalizations of well-known criteria are used for the examination of stability of the systems with retardation. The following result is formulated: Let be given the systems

(1) $$\frac{dx_i}{dt} = X_i(x_1,\ldots,x_n,x_1(t-h_{i1}(t)),\ldots x_n(t-h_{in}(t)),t)$$

and

(2) $$\frac{dx_i}{dt} = X_i(x_1,\ldots,x_n,x_1(t-h^*_{i1}(t)),\ldots,x_n(t-h^*_{in}(t)),t) + $$
$$+\varphi_i(x_1,\ldots,x_n,x_1(t-h^*_{i1}(t))\ldots,x_n(t-h^*_{i1}(t)),t).$$

Let the X_i be continuous in the neighborhood of $x1 = \ldots = x_n = 0$ and

Doklady Akad.Nauk 109. 400-463 (1956)

satisfy in x_i the Lipschitz condition with the constant L being independent of t. The $h_{ij}(t)$ are piecewise continuous, $0 \leq |h_{ij}(t)| < h$, where $h > 0$ is constant. The functions φ_i and h^*_{ij} satisfy the conditions

$$|\varphi_i(x_1,\dots,x_n,y_1,\dots y_n,t)| < \Delta_1(|x_1|+\dots+|x_n|+|y_1|+\dots+|y_n|)$$

$$|h_{ij}(t)-h^*_{ij}(t)| < \Delta_2, \qquad h^*_{ij} \geq 0.$$

Under these assumptions the following theorem is valid: If the solution $x_1 = \dots = x_n = 0$ of (1) is asymptotically stable, where

$$\sum_{i=1}^{n} |x_i(t)| \leq \sum \| x_{jo}(\theta)\| \cdot B \exp(-\alpha(t-t_o))$$

($\alpha > 0$, $B > 0$ - constant), then positive numbers Δ_1 and Δ_2 can be found such that the solution $x_1 = \dots = x_n$ of (2) is asymptotically stable too.

INSTITUTION: Polytechnical Institution, Ural.

KRASOVSKIY, N. N. Doc Phys-Math Sci -- (diss) "Certain Problems of the Theory of the Stability of Nonlinear Systems." Mos, 1957. 17 pp 22 cm. (Academy of Sciences USSR, Inst of Mechanics), 100 copies (KL, XXXXXX 25-57, 108)

KRASOVSKIY, N.N.

Smooth section of a dispersive dynamic system. Izv.vys.ucheb. zav.; mat. no.1:167-173 '57. (MIRA 12:10)

1. Ural'skiy politekhnicheskiy institut im. S.M.Kirova. (Differential equations)

103-11-2/10

AUTHOR: Krasovskiy, N. N., (Sverdlovsk)

TITLE: On the Theory of Optimum Control (K teorii optimal'nogo regulirovaniya).

PERIODICAL: Avtomatika i Telemekhanika, 1957, Vol. 18, Nr 11, pp. 960-970 (USSR)

ABSTRACT: Here it is assumed that that quantity is limited, the value of which, generally spoken, is determined by the behavior of the function $u_k(t)$ in the entire domain of the transition process. Apart from the case of a limitation of $u_k(t)$ in every moment t, the raising of such problems comprises also a number of other cases. Here the problem is dealt with in a manner that is different from that employed by Boltyanskiy-Gamkrelidze-Pontryagin (DAN SSSR, Vol. 60, Nr 1, 1956). Whenever there is agreement with the aforementioned work, the method employed here leads to the same results. Here it is assumed that the basic part of the system possesses given and unchanging parameters. First the problem of the optimum control is formulated. It is then investigated in the case of the limitation $g_T(u_1(\tau),\dots,u_r(\tau)) \leqslant N$. N denotes a constant quantity or function of the variable $x_{10},\dots\dots\dots,x_{no}$, T, g_T given functional which corresponds to

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On the Theory of Optimum Control

the limiting type with the function $u_k(t)$. g_T is a quantity, the value of which is computed according to a certain rule in accordance with the functions $u_1(\tau),\dots,u_r(\tau)$, which are given in the section $t_o \leqslant \tau \leqslant t_o+T$. The problem as such consists in determining a function $u_k^o(t)$ in such a manner that the trajectory $x_i(x_o, t_o \{u_k^o\}, t)$ reaches the point $x_i = \xi(t)$ within the shortest possible time $T = t-t_o$. Here in some cases continuous functions $u_k(t)$, which are limited by $g_T(u_1(\tau),\dots,u_r(\tau)) \leqslant N$, are admitted. Next, the problem of automatic control is investigated on the condition $|u_k(t)| \leqslant N$, where $t \geqslant t_o$, $k=1,\dots\dots,r$. In conclusion the approximated methods for the computation of optimum control functions $u_k^o(t)$ are dealt with. There are 11 Slavic references.

SUBMITTED: January 7, 1957

AVAILABLE: Dibrary of Congress

Card 2/2

KRASOVSKIY, N.N. (Sverdlovsk)

Stability in case of great initial perturbations. Prikl.mat. i mekh. 21 no.3:309-319 My-Je '57. (MIRA 10:10)
(Motion)

KRASOVSKIY, N.N.

40-5-8/20

AUTHOR: KRASOVSKIY, N.N. (Sverdlovsk)

TITLE: On a Problem of Optimum Control (Ob odnoy zadache optimal'nogo regulirovaniya).

PERIODICAL: Prikladnaya Mat. i Mekh., 1957, Vol. 21, Nr 5, pp. 670-677 (USSR)

ABSTRACT: For a system of n difference equations the following problem is considered: Let an initial point of the variables be given in the phase space and a sequence of points after certain time intervals. Such a control variable is to be sought that the sequence of the points converges to a presupposed point, and this in a number of steps as small as possible. Here certain restrictive conditions must be required for the control variables.

Similar problems have been already investigated for the behavior of systems of differential equations. An aim of the present paper is the clarification of the transition of the solution of a difference equation to the solution of a differential equation under continuous diminution of the step width. In this way an approximative method for calculating the solutions of differential equations can be found. The method given by the author consists of several simple operations, the number of which, however, extraordinarily increases with the degree

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KRASOVSKIY N.N.

AUTHOR: Germaidze, V.Ye. and Krasovskiy, N.N. 40-21-6-5/18
(Sverdlovsk)

TITLE: Stability Under Constantly Acting Disturbances (Ob ustoychivosti pri postoyanno deystvuyushchikh vozmushcheniyakh)

PERIODICAL: Prikladnaya Matematika i Mekhanika, 1957, Vol 21, Nr 6, pp 769-774 (USSR)

ABSTRACT: The authors investigate the behavior of the solutions of different differential equations. The initial equations are essentially systems of differential equations of first order in which on the right side there stand arbitrary functions of the variables and of the time. Starting from the solutions of a certain fundamental system conclusions on the solutions of disturbed systems are obtained. Several theorems of the following kind are proved : If the zero solution of the shortened initial equation is uniformly asymptotically stable, then it is stable too if there exist constantly acting external disturbances which are bounded in the mean. The proof of these and of similar theorems is carried out according to Lyapunov's method. Surpassing the investigations usual till now the authors still deal with systems of equations, the variables

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Stability Under Constantly Acting Disturbances 40-21-6-5/18

of which possess delayed arguments. Even for such systems of equations corresponding theorems like that one mentioned above can be proved. The paper is particularly interesting, since in its third section a proof for the frequently applied and extraordinarily useful method of the harmonic balance is given. A general theorem is derived which represents a mathematical foundation to this method heuristically applied till now. There are 11 references, 9 of which are Soviet, and 2 American.

SUBMITTED: July 18, 1957

AVAILABLE: Library of Congress

1. Differential equations-Analysis

Card 2/2

KRASOVSKIY, N. N.

20-2-5/60

AUTHOR: Krasovskiy, N. N.

TITLE: On the Periodic Solutions of Differential Equations With Time Retardation (O periodicheskikh resheniyakh differentsial'nykh uravneniy s zapazdyvaniyem vremeni)

PERIODICAL: Doklady Akademii Nauk SSSR, 1957, Vol. 114, Nr 2, pp.252-255 (USSR)

ABSTRACT: In a complicated generating system the existence and uniqueness theorems (or the theorems of isolation) of the generatingoscillations are of importance. The present report shall show the possibility of the solution of such problems for equations with a time retardation. Similar as in the case of ordinary equations the method of the Lyapunov functions is employed here. Only the rough case of the asymptotic stability is examined here, but for systems of fairly general form. The author finds the sufficient conditions for the existence and for the uniqueness of the periodic solution for a generating system. The author investigated the following system of equations (1):

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On the Periodic Solutions of Differential Equations With Time Retardation

$$dx_i/dt = \sum_{j=1}^{n} p_{ij}(t)x_j(t - h_{ij}(t)) +$$

$$+ \varphi_i\,(x_1(t - h_{i1}(t)), \ldots, x_n(t-h_{in}(t)), t) + f_i(t);$$

$$\varphi_i(0, \ldots, 0, t) = 0; \; (i = 1, 2, \ldots, n).$$

In this connection $p_{ij}(t)$, $\varphi_i(x_1, \ldots, x_n, t)$, $f_i(t)$ signify continuous functions and $h_{ij}(t)$ piece-by-piece periodic functions of time with the period T, and $0 \leqslant h_{ij}(t) \leqslant H$ applies. The equations just given evidently also include systems of equations with finite differences. The author also investigates the system of equations (2):

$$dy_i/dt = \sum_{j=1}^{n} q_{ij}(t)y_j(t-g_{ij}(t)); \; (i = 1, 2, \ldots, n),$$ where

$q_{ij}(t)$ signify continuous functions and $g_{ij}(t) \geqslant 0$ piece-by-piece periodic functions. The following theorems are given and proved:

Theorem 1: When the solution $y_1 = \ldots = y_n = 0$ of the equations (2) is asymptotically stable, positive numbers δ, γ and L

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On the Periodic Solutions of Differential Equations With Time Retardation

can be given so that, when the inequations

$$|p_{ij}(t)-q_{ij}(t)| < \delta, \ |h_{ij}(t)- \ _{ij}(t)| < \gamma, \ |\varphi_i(x_1'', \ldots, x_n'', t) = \varphi_i(x_1', \ldots, x_n', t)| < L\sum_{j=1}^{n} | x_j''-x_j' |$$

, are satisfied, the system of equations (1) has a uniquely periodic solution which is asymptotically stable in the Lyapunovan sense.

Theorem 2: When the solution $y_1 = \ldots = y_n = 0$ of the system of equations (2) is asymptotically stable, the solution of the equations

$$dx_i/dt = \sum_{j=1}^{n} (q_{ij}(t) + \mu p_{ij}(t))x_j(t- \left[g_{ij}(t)+ \mu h_{ij}(t)\right]) + f_i(t) + \mu\varphi_i(x_1(t- \left[g_{ij}(t) + \mu h_{ij}(t)\right]), \ldots, x_n(t- \left[g_{ij}(t) + \mu h_{ij}(t)\right]), t),$$

is stable with regard to the parameter μ.

There are 12 references, 10 of which are Slavic.

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On the Periodic Solutions of Differential Equations With Time Retardation

ASSOCIATION: Ural Polytechnical Institute imeni S. M. Kirov (Ural'skiy politekhnicheskiy institut im. S. M. Kirova)

PRESENTED: December 10, 1956, by I. G. Petrovskiy, Academician

SUBMITTED: April 16, 1956

AVAILABLE: Library of Congress

Card 4/4

AUTHOR: Krasovskiy, N.N. 20-119-3-9/65

TITLE: On the Stability of Quasilinear Systems With Afterworking (Ob ustoychivosti kvazilineynykh sistem s posledeystviyem)

PERIODICAL: Doklady Akademii Nauk, 1958, Vol 119, Nr 3, pp 435-438 (USSR)

ABSTRACT: Well-known stability theorems are transferred to systems

$$(1)\ \frac{dx}{dt} = X\left[x(t+\vartheta),t\right] + R\left[x(t+\vartheta),t\right] \quad (-h \leqslant \vartheta \leqslant 0),$$

where x is an element of the Banach space B; X and R are operators on continuous curves $x(\vartheta)$ $(-h \leqslant \vartheta \leqslant 0)$ which map these curves into B, $t \geqslant 0$.
The space of the continuous curves $x(\vartheta)$ with the norm $\|x(\cdot)\| = \sup\|x(\vartheta)\|$, $-h \leqslant \vartheta \leqslant 0$, is denoted with $B(\cdot)$, its elements with $x(\cdot)$. The linear equation

$\frac{dx}{dt} = X(x(t+\vartheta))$, $x \in B$, $\|X\| = L$, is equivalent to the equation

$\left.\frac{dx(t,\cdot)}{dt}\right|_{dt=+0} = A\,x(t,\cdot)$, $x(t,\cdot) \in B(\cdot)$, whereby A is the un-

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On the Stability of Quasilinear Systems With Afterworking 20-119-3-9/65

bounded operator $A\, x(\cdot) = y(\cdot) = \begin{cases} y(\vartheta) = \frac{dx}{d\vartheta} & \text{for } -h \leqslant \vartheta < 0 \\ y(0) = X[x(\vartheta)] \end{cases}$

Theorem: If the spectrum $\{\lambda_\sigma\}$ of A satisfies the condition

(2) $\quad \operatorname{Re} \lambda_\sigma \leqslant -\gamma \quad (\gamma > 0)$,

then there exists a functional $v[x(\cdot)]$ which satisfies the following estimations:

$$c_1 \|x(\cdot)\|^k \leqslant v[x(\cdot)] \leqslant c_2 \|x(\cdot)\|^k$$

$$\limsup_{\Delta t \to +0} \frac{v[x(t+\Delta t, \cdot)] - v[x(t, \cdot)]}{\Delta t} \leqslant -c_3 \|x(\cdot)\|^k$$

$$\left| v[x''(\cdot)] - v[x'(\cdot)] \right| \leqslant c_4 \|x''(\cdot) - x'(\cdot)\| \sup(\|x''(\cdot)\|^{k-1}, \|x'(\cdot)\|^{k-1}),$$

where $k > 0$ is an arbitrary fixed integer, $c_i > 0$ are constants.

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On the Stability of Quasilinear Systems With Afterworking 20-119-3-9/65

With the aid of this theorem the author shows: 1.) If (2) is satisfied, then there exists a constant $a > 0$, so that the solution $x(t,\cdot) \equiv \theta(\cdot)$ of (1) is asymptotically stable, as soon as $\| R[x(\vartheta),t] \| \leqslant a \| x(\cdot) \|$. 2.) A more complicated criterion in which a lower bound is given for the characteristic numbers of the solution. There are 13 references, 9 of which are Soviet, 3 American, and 1 Hungarian.

ASSOCIATION: Ural'skiy politekhnicheskiy institut imeni S.M.Kirova (Ural Polytechnical Institute imeni S.M. Kirov)

PRESENTED: July 22, 1957, by I.G. Petrovskiy, Academician

SUBMITTED: April 8, 1957

Card 3/3

16(1) PHASE I BOOK EXPLOITATION SOV/3125

Krasovskiy, Nikolay Nikolayevich

Nekotoryye zadachi teorii ustoychivosti dvizheniya (Some Problems in the Theory of Stability of Motion) Moscow, Fizmatgiz, 1959. 211 p. 5,000 copies printed.

Ed.: G.I. Fel'dman; Tech. Ed.: S.N. Akhlamov.

PURPOSE: This book is intended for mathematicians and physicists.

COVERAGE: The author examines certain problems of the theory of the stability of the solutions of nonlinear systems of ordinary differential equations. The book deals primarly with the solution of general theoretical problems on the possibilities of Lyapunov's method and on certain basic procedures for applying the method to the study of specific stability problems. The book is based mostly on works of the author published in various journals, but formulations and proofs have been revised and supplemented. The works of some Soviet and non-Soviet authors were also used in preparing the book. The author

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thanks N.G. Chetayev for his advice and criticims concerning the work and Ye.A. Barbashin, M.A. Ayzerman, N.P. Yerugin, Ya. Kurtsveyl', A.M. Letov, I.G. Malkin, V.V. Nemytskiy, B.S. Razumikhin for taking part in the discussion of the work. There are 141 references: 129 Soviet, 10 English, 1 German, and 1 Spanish.

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APPROVED FOR RELEASE: Monday, July 31, 2000 CIA-RDP86-00513R000826210(

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05257

16(1) SOV/140-59-5-13/25

AUTHOR: Krasovskiy, N.N.

TITLE: On Optimal Control in Nonlinear Systems

PERIODICAL: Izvestiya vysshikh uchebnykh zavedeniy. Matematika, 1959, Nr 5, pp 122-130 (USSR)

ABSTRACT: The method for the determination of the optimal control proposed by L.S.Pontryagin [Ref 1] has an essential want for the practical application: The conditions of optimality obtained from the maximum principle lead to a system of differential equations and the integration constants appearing at the solution have to be determined so that the optimal trajectory runs into the origin after a certain time; but an effective method for the solution of this boundary value problem is missing.
The author proposes an approximate method for which this difficulty can be avoided by the introduction of a parameter. The equations which describe the dependence of the optimal trajectories from the parameter are extraordinarily difficult in the nonlinear case and can be treated only numerically.
There are 4 Soviet references.

ASSOCIATION: Ural'skiy gosudarstvennyy universitet imeni A.M.Gor'kogo (Urals State University imeni A.M.Gor'kiy)

SUBMITTED: April 21, 1959

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APPROVED FOR RELEASE: Monday, July 31, 2000 CIA-RDP86-00513R000826210(

06310
16(1) SOV/140-59-6-11/29

AUTHOR: Krasovskiy, N.N.

TITLE: On the Problem of the Existence of Optimal Trajectories

PERIODICAL: Izvestiya vysshikh uchebnykh zavedeniy. Matematika, 1959, Nr 6, pp 81-87 (USSR)

ABSTRACT: The author considers control systems described by

(0.1) $\frac{dx}{dt} = f(x) + bu$,

where $f(x)$ has continuous derivatives up to the order n-1 and $u(t) = \{u_1(t),\dots,u_n(t)\}$ is the so-called controlling device. For given initial conditions x_0 the author seeks a $u^0(t)$, $|u^0(t)| \leqslant 1$ so that the motion along the trajectory $x(t)=x(x_0,t,u_0)$ from x_0 to 0 is carried out in a shortest time. Under the assumption that the matrix A $\left(\{A\}_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right)_{x=0}\right)$ in the system of the first approximation

(0.3) $\frac{dx}{dt} = Ax$

in the point x=0 satisfies the conditions usual for linear

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On the Problem of the Existence of Optimal Trajectories

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systems, the existence of a domain of optimal attraction in the neighborhood of $x=0$ is proved. The result is used for proving the existence of optimal trajectories for nonlinear systems

$$\frac{dx}{dt} = f(x) \tag{0.4}$$

the trivial solution of which is asymptotically stable. Three theorems are formulated.

There are 12 references, 11 of which are Soviet, and 1 American.

ASSOCIATION: Ural'skiy politekhnicheskiy institut imeni S.M.Kirova (Ural Polytechnical Institute imeni S.M.Kirov)

SUBMITTED: March 3, 1959

Card 2/2

KRASOVSKIY, N.N. (Sverdlovsk)

Sufficient conditions for optimity. Prikl. mat. i mekh. 23 no.3:592-594 My-Je '59. (MIRA 12:5)
(Automatic control)

16(1) SOV/20-126-2-11/64

AUTHOR: Krasovskiy, N.N.

TITLE: On the Theory of Optimal Control of Nonlinear Systems of Second Order

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 126, Nr 2, pp 267-270 (USSR)

ABSTRACT: The author considers the system

(1) $$\frac{dx}{dt} = f(x,t) + q(t)\eta ,$$

where x,f,q are two-dimensional vectors. For $t \in (t_\alpha, t_{\alpha+1})$ the functions f have continuous partial derivatives $\partial f_i/\partial x_j$ bounded with respect to their absolute value uniformly by L; for $t = t_\alpha$, f and $\partial f_i/\partial x_j$ may have discontinuities of first kind. The motion described by (1) begins in the moment $t_0 = 0$ in the point $x = x_0$. The author seeks a piecewise smooth function $\eta_0(t)$, $|\eta_0(t)| \leq 1$, so that the point x(t) reaches the point x = 0 in the shortest time. As in the paper of Pontryagin [Ref 1] the author introduces notions of the admissible controlling, of the optimal trajectories etc. The author proves the correctness of the given

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On the Theory of Optimal Control of Nonlinear Systems of Second Order SOV/20-126-2-11/64

problem and he extends the maximum principle of Pontryagin to the considered instationary case. Basing on the general investigations of Pontryagin and results of the functional analysis and the qualitative theory of differential equations the author formulates six theorems on the existence of optimal trajectories, on necessary and sufficient conditions for the optimality and on a method for the successive approximation of the optimal trajectory.
There are 8 Soviet references.

ASSOCIATION: Ural'skiy politekhnicheskiy institut imeni S.M.Kirova (Ural Polytechnical Institute imeni S.M.Kirov)

PRESENTED: January 22, 1959, by L.S.Pontryagin, Academician

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KRASOVSKIY, A. A.

Report to be presented at the 1st Intl Congress of the Intl Federation of Automatic Control, 25 Jun-5 Jul 1960, Moscow, USSR.

BYKHOVSKIY, M. L. - "Ultra stability in electronic calculating devices
in the solution of nonlinear equations in indefinite form"
CHELYUSTKIN, A. B. - "Use of calculating devices in systems for
the automatic control of rolling mills"
CHICHINADZE, V. K. - "Concerning some problems of the organization of
self-adjusting and self-teaching systems of automatic control,
based on principles of random search"
DAVYDOV, E. I. - "Development of automatic control systems for boiler
units"
DUDNIKOV, Ye. G. - "Determination of optimum adjustments of industrial
automatic regulation systems according to initial data obtained from
experience"
DUB'YE, A. I., and [illegible], E. N. - "Methods of organizing Lyapunov
functions in the theory of nonlinear regulating systems"
[illegible], I. S. - "Balanced regulation and inter-communications of
a multi-motor electric drive and technology in continuous rolling
mills"
FEL'DBAUM, A. A. - "Problems of statistical theory of automatic
optimization systems"
FETGIN, V. L. - "Automation of a reversible cold rolling mill for
nonferrous metals"
FILIPPOV, A. F. - "Application of the theory of differential equations
with a discontinuous right side to nonlinear problems of automatic
regulation"
GAVRILOV, M.A. - "Structural surplus and operational reliability of
relay devices"
GANKIN, M. Z. - "Automation of irrigation systems"
[illegible], G. E., [illegible], V. E., [illegible], M. P., [illegible], L. R.,
and [illegible], E. S. - "Power regulation of disturbance and problems of
the stability of electric power systems"
GINZBURG, S. A. - "Logical method of synthesis of functional converters"
IL'IN, V. A. - "Methods of transmission of information and the structure
of telemechanical systems for dispersed structures"
ILYUSOV, V. L., and LITSKIY (fnu) - "The code-impulse system of tele-
measurement for dispatched operations of trunk-line gas pipe lines"
IVAKHNENKO, A.G. - "Concerning the application of the theory of combined
regulation systems for cybernetic adaptation systems"
KARANDEYEV, K. B., and SHTAMBERGER, G. A. - "A quasi-equilibrated bridge
as an element in a system of automatic control"
KAZAKEVICH, V. V. - "Concerning the process of extra regulation of inert
objects in the presence of disturbance"
KAZAKOV, I. E. - "Some problems of the theory of statistical linearization
and its application"
KIPIN, F. M. - "Some problems of the theory of impulse systems with
time selectors"
[illegible], A. E., BOLKHOVITIN, S. V., VOLKOBOYNIKOV, L.M., IOFFE, D. M.,
POLYAN, E. P., POPOV, B. P., SLAVUTSKIY, Ya. L., SYSIN, A. Ya., and
YAKOBSON, Ya. S. - "The problem of bioelectric control"
KOLOMIYETS, B.T. - "New types of photo resistances and their field of use"
KOROBKO, M. I., MIKHAYLOV, B. G., and SHCHEGLOV, K. A. - "System of
automatic control and regulation of blast distribution in the tuyeres
of blast furnace"
KOROBOCHKIN, B. L. - "Investigation of the dynamics of the hydraulic
duct of a copying lathe"
KRASOVSKIY, A. A. - "Dynamics of continuous systems of automatic
regulation with extra self-adjustment of corrective devices"
~~KRASOVSKIY, N. N.~~ - "Concerning the selection of parameters of
optimum stability systems"
KUKHTENKO, A. I. - "The dynamics of devices imitating living organisms"
KULEBAKIN, V. S. - "The invariant theory of automatic regulation and
control systems"
LANIN, L. D. - "Pneumatic calculating devices as a means of insuring the
reliability of complex automation systems"
LAZAREV, V. [illegible] and [illegible], P.P. - "Mechanization of processes
of analysis and synthesis of the structure of relay devices"

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AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: Optimum Control With Random Perturbations

PERIODICAL: Prikladnaya matematika i mekhanika, 1960, Vol 24, Nr 1, pp 64-79 (USSR)

ABSTRACT: The optimum control of a system described by

$$\frac{dx}{dt} = Ax + Bu + c\eta \qquad (1.1)$$

is studied under the condition that the expected time of damping of the transient process be a minimum. Here x is an n-dimensional vector in phase space, A and B are n-by-n matrices, $\eta(t)$ is a random scalar function, and u is an n-dimensional steering function. For given initial conditions x_0, η_0, t_0,

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it is required to determine the steering function u^o which guarantees that the damping time of the transient process $x(x_o, \eta_o, t_o, t, \eta, u)$ be a minimum. The author reformulates the problem as follows. He defines a set of operators $u[t, g]$ which is denoted by U_t for $t \geq t_o$ which compares vectors u with realizations of the random function $\eta(t)$. The set U_t is restricted so that $\| u[t, g] \| \leq 1$ for t in the interval $t_o, \infty)$ and such that

$$T[U_t, k, \varepsilon, x_0, \eta_0, t_0] = \int_{t_0}^{\infty} p[U_t, k, \varepsilon, x_0, \eta_0, t_0, t]\, dt < \infty \qquad (1.3)$$

Here, $p[U_t, k, \epsilon, x_o, \eta_o, t_o, t]$ denotes the probability that

$$\| x(x_0, \eta_0, t_0, t, \eta, u) \|_k > \varepsilon \qquad (1.4)$$

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holds for $t \geq t_0$ along a random solution of the given system of equations generated by the random functions $\eta(t)$ and $u(t)$. (The norms used are defined by $\| y \| = \sqrt{y_1^2 + \ldots + y_n^2}$ and $\| y \|_k = \sqrt{y_1^2 + \ldots + y_k^2}$ where $k \leq n$.) The "optimum problem" is then the problem of determining an admissible steering function $\overset{o}{U_t}$ from the considered set such that

$$T\,[\overset{o}{U_t}] = \min\,(T\,[U_t]\,).$$

The author first proves an existence theorem for the special case given by

$$x_1^{(n)} + a_1 x_1^{(n-1)} + \ldots + a_n x_1 = u_1(t) + \eta_1(t) \qquad (2.1)$$

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with k and ϵ taken as n and 0 respectively. The function $\eta(t)$ is assumed to be piecewise constant on certain half-closed intervals, finite in number; the constant values η_l are assumed to satisfy:

$$|\eta_l| \leq q < 1 \qquad (l = 1, \ldots, m) \tag{2.2}$$

Furthermore, it is assumed that the roots of the characteristic equation $|A-\lambda I| = 0$ have negative real parts. The admissible realizations of u(t) are considered to be piecewise smooth functions having finite jumps at isolated values of t, and the admissible solutions x(t), those continuous functions satisfying (2.1) except at the stated points of discontinuity. The author shows that an admissible steering function U_t exists for any choice of initial conditions x_0, η_0, t_0, and using this he proves that an optimum steering

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function U_t^o exists for any choice of the initial data; i.e., there exists an admissible U_t^o satisfying the condition

$$T[U_t^o] = T^o \qquad (2.9)$$

where T^o is the limit of a certain sequence of $T\left[U_t^{(k)}\right]$ as $k \to \infty$ corresponding to a sequence of the existing admissible U_t. The author then considers the equation

$$x_1^{(n)} + a_1 x^{(n-1)} + \ldots + a_n x_1 = u(t) + \eta(t) + \xi\delta(t - t_0) \qquad (3.1)$$
$$(x_i = x_1^{(i-1)};\ i = 1, \ldots, n)$$

where ξ is a random independent variable with a small dispersion σ^2. It is assumed that the mathematical expectations of ξ and ξ^2 are zero and σ^2, respectively. The author states (without proof) that the optimum conditions for the first problem can be obtained from this problem by letting ξ and ϵ^2 tend to zero. He then deduces a necessary condition for the optimality of the steering function with the same choice of $\eta(t)$ as in the previous problem. He notes that standard approximation methods (e.g., method of steepest descent) can be used though with considerable difficulty to solve the variational problem. The author also describes, by proving several theorems, a generalization of the Lyapunov function and the application of the second method of Lyapunov to optimum problems: the existence of admissible optimum steering functions and the construction of such functions. As illustrations, the optimum control for random noise is discussed and an approximate graphical method is described for constructing the optimum Lyapunov function for a system of equations of the second order corresponding

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to the equation

$$\ddot{x} + a_1\dot{x} + a_2 x = u_1 + \eta(t) \qquad (6.2)$$

and the conditions $|u_1| \leq 1$. There are 18 references, 16 Soviet, 2 U.S. The U.S. references are: Lasalle, J. P., Time Optimal Control Systems, Proc. of the National Acad. of Sci., Vol 45, Nr 4, (1959); Bellman R., Dynamic Programming and Stochastic Control Processes, Information and Control, Vol 1, pp 228-239 (1958).

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AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: On Approximative Calculation of the Optimum Control With a Direct Method

PERIODICAL: Prikladnaya matematika i mekhanika, 1960, Vol. 24, No. 2, pp. 271-276

TEXT: Let the system

$$\frac{dx}{dt} = Ax + bu \tag{1.1}$$

be considered, where $x = (x_1, x_2, \ldots, x_n)$ is the image point in the phase space, A an $n \times n$ matrix, b a vector characterizing the structure of the automatic control system and $u = u(t)$ the control function which satisfies the additional condition

$$|u(t)| \leq 1. \tag{1.2}$$

The optimum control $u^{\circ}(t)$ is sought for which the image point arrives from the initial position $t = t_o = 0$, $x = x_o$ at the final position $x = 0$ in the shortest time T°. The principal solution of this problem is given by the maximum principle (Ref.2). The real calculation of

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$u^{o}(t)$ and of the optimum trajectory $x^{o}(t) = x(x_o, t, u^{o})$, however, raises great difficulties. The author proposes the following approximation method.

Let $U_{\varepsilon}[x,t]$ be a continuously differentiable function with respect to x_i with the properties

(1.3) $0 \leqq U_{\varepsilon} \leqq 1$ for all t, x; $U_{\varepsilon}[0,t] = 0$; $U_{\varepsilon}[x,t] = 0$ for $t > \tau_{\varepsilon}$

(1.4) $U_{\varepsilon}[x,t] \geqq q(\varepsilon)$ for $\|x\| \geqq \varepsilon > 0$, $0 \leqq t \leqq \theta_{\varepsilon}$, $\|x\| = \sqrt{\sum x_i^2}$

(1.5) $\lim q(\varepsilon) = 1$, $\lim \theta_{\varepsilon} = \infty$, $\lim \tau_{\varepsilon} = \infty$ for $\varepsilon \to 0$

Auxiliary problem A_{ε}: For given $x = x_o$, $t_o = 0$ an admissible control $u_{\varepsilon}^{o}(t)$ is to be determined so that

(1.6) $$T_{\varepsilon}^{o} = \int_0^{\infty} U_{\varepsilon}[x(x_o, t, u_{\varepsilon}^{o}), t]\, dt = \min$$

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Auxiliaryproblem $A_{\varepsilon K}$: For given $x = x_0, t_0 = 0$ the control $u^{\circ}_{\varepsilon K}(t)$ is to be determined so that

$$(1.7)\quad T^{\circ}_{\varepsilon K} = \int_0^{\infty} \left(U_{\varepsilon}\left[x(x_0, t, u^{\circ}_{\varepsilon K}) \; t\right] + \left[u^{\circ}_{\varepsilon K}(t)\right]^{2K}\right) dt - T^{\circ} = \min,$$

where k is a natural number and $u^{\circ}_{\varepsilon K}$ does not satisfy (1.2).

Theorem 2.1: It is

$$(2.1)\quad \lim T^{\circ}_{\varepsilon} = T^{\circ} \qquad \text{for } \varepsilon \to 0$$

$$(2.2)\quad \lim T^{\circ}_{\varepsilon K} = T^{\circ} \qquad \text{for } \varepsilon \to 0,\ k \to \infty$$

where the functions $u^{\circ}_{\varepsilon}(t)$, $u^{\circ}_{\varepsilon K}(t)$ for $\varepsilon \to 0$, $k \to \infty$ converge in the mean to the function $u^{\circ}(t)$ on $[0, T^{\circ}]$ (in L_2).

Theorem 3.1: $u^{\circ}_{\varepsilon}(t)$ satisfies the condition

$$(3.1)\quad u^{\circ}_{\varepsilon}(t)\left(\Psi^{\circ}(t) \cdot b\right) = \max,$$

where $\Psi^{\circ}(t)$ is the particular solution of

$$(3.2)\quad \frac{d\Psi}{dt} = -A'\Psi + \eta(t), \quad \eta_i(t) = \frac{\partial U_{\varepsilon}\left[x^{\circ}_{\varepsilon}(t), t\right]}{\partial x_i}$$

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and A' denotes the matrix transposed to A.
The theorem show that the initial problem can be reduced to an ordinary wariational problem. Since, however, the use of the conditions (3.1), (3.2) is difficult the author recommends the following direct way for the probelm $A_{\varepsilon\kappa}$: Take

$$U_\varepsilon = 1 - \exp\left(-\frac{x^2}{2\varepsilon}\right) \text{ for } t \in [0, \tau_\varepsilon] \quad , U_\varepsilon = 0 \text{ for } t > \tau_\varepsilon$$

and seek the optimum control in the form

$$(4.1) \quad u(t) = a_1 \sin\frac{t}{\tau_\varepsilon} + \dots + a_\ell \sin\frac{\ell t}{\tau_\varepsilon}$$

if (4.1) is substituted into (1.7) and into the general solution

$$(2.11) \qquad x(t) = F(t)x_0 + \int^t F(t)\, F^{-1}(\tau)\, bu(t)\, d\tau$$

of (1.1), then the problem

$$(4.2) \min F(a_1, \dots, a_\ell) = \min \left[\int_0^{\tau_\varepsilon} \left(U_\varepsilon[x(x_0, t, u), t] + u^{2\kappa}(t)\right) dt\right]$$ K

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is obtained, the solution $a_1^0, \dots a_l^0$ of which yields a control

$$u'(t) = a_1^0 \sin\frac{t}{\tau_1} + \dots + a_l^0 \sin\frac{t}{\tau_l}$$

which for sufficiently small ε and sufficiently great k and l is arbitrarily near to $u^0(t)$.

L. J. Rozonoer is mentioned in the paper.

There are 10 references: 9 Soviet and 1 American.

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AUTHORS: Kats, I. Ya., and Krasovskiy, N. N. (Sverdlovsk)

TITLE: On the Stability of Systems With Random Parameters

PERIODICAL: Prikladnaya matematika i mekhanika, 1960, Vol. 24, No. 5, pp. 809-823

TEXT: The authors consider the equations of the disturbed motion

(1.1) $dx/dt = f(x,t,y(t))$,

where $x = \{x_1, \ldots, x_n\}$, $f = \{f_1, \ldots, f_n\}$, the f_i are continuous with respect to all arguments, and in

(1.3) $\|x\| < H, \quad t \geqslant t_0$,

where $\|x\| = \max(|x_1|, \ldots, |x_n|)$ it holds:

(1.2) $|f_i(x'', t, y, (t)) - f_i(x', t, y(t))| \leqslant L \|x'' - x'$

Here $y(t)$ is a homogeneous Markov chain with a finite number of states, i.e. in every moment, $y(t)$ can assume one of the values y_i out of a finite set of values $Y(y_1, \ldots, y_r)$, where the probability $p_{ij}(\Delta t)$ of the

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change $y_i \to y_j$ in the time Δt satisfies the condition

$$(1.4) \qquad p_{ij}(\Delta t) = \alpha_{ij} \Delta t + o(\Delta t) \qquad (i \neq j) \qquad (\alpha_{ij} = \text{const}),$$

where $o(\Delta t)$ is infinitely small of higher order than Δt. It is assumed that $y_i = i$ $(i=1,2,\ldots r)$ and that

$$(1.5) \qquad f_i(0,t,y(t)) \equiv 0 \qquad (y \in Y,\ t \geq 0).$$

A random vector function $\{x(x_0,t_0,y_0;t), y(t_0,y_0;t)\}$ the realizations $\{x^{(p)}(x_0,t_0,y_0;t), y^{(p)}(t_0,y_0;t)\}$ of which satisfy (1.1) is called a solution of (1.1).

The authors investigate the probability stability (cf.(Ref.5)) and the asymptotic probability stability of the solution $x = 0$ of (1.1). The conditions of stability are given in terms of Lyapunov functions. A function $v(x,t,y)$ is called positive definite if $v(x,t,y) \geq w(x)$ for all $y \in Y$, $t \geq t_0$, where $w(x)$ is positive definite in the sense of

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Lyapunov; $v(x,t,y)$ is said to be of constant sign if in (1.3) it cannot assume values of a distinct sign. A function $v(x,t,y)$ admits an infinitely small least upper bound if there exists a continuous $W(x)$ so that $v(x,t,y) \leq W(x)$, $W(0) = 0$ for $\|x\| < H$, $t \geq t_0$, $y \in Y$.

A function $v(x,t,y)$ admits an infinitely large greatest lower bound in $\|x\| < H$ if $w(x)$ (cf. above) satisfies the condition $\lim w(x) = \infty$ for $\|x\| \to H$. Let $M[\psi(\alpha_1,\ldots,\alpha_n); \alpha_1,\ldots,\alpha_n/\beta]$ denote the mathematical expectation of the function $\psi(\alpha_1,\ldots,\alpha_n)$ of the random variable $\alpha_1,\ldots,\alpha_n$ under the conditions β. Let $M[v] = M[v(x(t),t,y(t));$ $x(t)\ y(t)/x(t) = \xi, y(t) = \eta]$, where $\{x(t), y(t)\}$ is the solution generated for $t = \tau$ by the initial conditions $x = \xi$, $y = \eta$, be the mathematical expectation of the random function $v(x(\xi,\tau,\eta;t),t,y(\tau,\eta;t))$ for $t \geq \tau$. The limit value

$$(2.1)\quad \frac{dM[v]}{dt} = \lim_{t \to \tau+0} \frac{1}{t-\tau}\{M[v(x(t),t,y(t)); x(t), y(t)/x(\tau)=\xi, y(\tau)=\eta] - v(\xi,\tau,\eta)\}$$

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is denoted as the derivative $\frac{dM[v]}{dt}$ of v for (1.1) in $x = \xi$, $y = \eta$, $t = t$.
Theorem 3.1: If for (1.1) a positive definite function v(x,t,y) can be given so that $\frac{dM[v]}{dt}$ for (1.1) is of constant negative sign then the solution x = 0 is probability stable.
Theorem 3.2: If for (1.1) there exists a positive definite v(x,t,y) which admits an infinitely small least upper bound, and the derivative of which for (1.1) is negative definite in (1.3) then for every number $p(H) < 1$ there exists a number H_0 so that the solution x = 0 of (1.1) is p(H)-asymptotically stable with respect to the disturbances out of the region

(1.9) $$\|x_0\| < H_0$$

(A solution is called p(H)-asymptotically stable with respect to initial disturbances of (1.9) if it is probability stable and besides $\lim p_t(\|x\| \ldots) \geq 1 - p(H)$ for $t \to \infty$, where $p_t(\ldots)$ is the probability that for $t > t_0$ it holds $\|x\| < \ldots$, where $y_0 \ldots Y$)

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For the case $H = \infty$ the authors obtain results corresponding to those of (Ref.4).
Then the authors consider systems

(5.1) $dx/dt = A(t,y)x+R(x,t,y)$,

where the elements of the matrix $A(t,y)$ for all $y \in Y$ are continuous bounded functions of the time, while with respect to the $R_1(x,t,y)$ it is assumed that in (1.3) and for all $y \in Y$ it holds

(5.2) $|R_i(x,t,y)| \leq \gamma \|x\|_2^2$ (γ = const > 0),

where $\|x\|_2 = \sqrt{x_1^2+\ldots+x_n^2}$.
Beside of (5.1) the authors consider the system of the first approximation

(5.3) $dx/dt = A(t,y)x$.

Theorem 5.1: If the solution of (5.3) is exponentially stable in the mean then the corresponding solution of (5.1) is probability stable; furthermore: for every p(H) the solution x = 0 is p(H)-asymptotically stable for arbitrary R(x,t,y) which in (1.3) satisfy the condition (5.2)
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if γ is sufficiently small (the solution $x = 0$ of (1.1) is called exponentially stable in the mean if for arbitrary initial conditions from (1.3) there exist constants B and α so that for all $t \geq t_0$ it holds

(4.5) $M\left[\|x(t)\|_2^2 ; x(t)/x_0, y_0\right] \leq B\|x_0\|_2^2 \exp(-\alpha(t-t_0))$).

The authors consider the stationary linear system

(6.1) $dx/dt = A(y)x$

Theorem 6.1: If the solution $x = 0$ of (6.1) is asymptotically stable in the mean (i.e. stable in the quadratic mean (cf.(Ref.5)) and besides for all solutions with the initial conditions $\|x_0\|_2 < H_0$ satisfying the condition $\lim M\{\|x(t)\|_2^2\} = 0$ for $t \to \infty$), then for every given positive definite form $w(x,y)$ there exists one and only one form $v(x,y)$ of the same order which satisfies the equation

(6.2) $dM\{v\}/dt = -w(x,y)$;

this form is always positive definite

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Theorem 6.2: If the solution x = 0 of the system (6.1) is asymptotically stable in the mean then the corresponding solution of the equation

(6.11) $dx/dt = A(y)x+R(x,t,y)$

is p(H)-asymptotically stable if (5.2) is satisfied, and χ is sufficiently small.

Finally the stability for random continuously acting disturbances is considered briefly.

There are 11 references: 7 Soviet and 4 American.

[Abstracter's note: (Ref.4) concerns I.E.Bertram and P.E.Sarachik, Proc. Int.Symp. on Circuit and Information Theory, 1959 . (Ref.5) concerns J.Doob, Stochastic Processes.]

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16.8000 (1031, 1132, 1329)

AUTHOR: Krasovskiy, N.N. (USSR)

TITLE: On the choice of parameters in stable optimal systems

SOURCE: IFAC, 1st Congress, Moscow 1960. Teoriya diskretnykh, optimal'nykh i samonastraivayushchikhsya sistem. Trudy, v. 2, 1961, 482 - 490

TEXT: Some results are given, relating to the choice of optimum controllers subject to certain restrictions. The system is described by the equations

$$\frac{dx}{dt} = f(x, t) + Bu, \qquad (1)$$

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where x and f are n-dimensional vectors, u is the r-dimensional controller-vector, B is a matrix. The functional $g_T[u]$ is given which determines the restrictions on the allowed controller, viz.

$$g_T[u] \leq 1. \qquad (2)$$

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It is necessary to determine the allowed optimal controller (a.o.c.) for which system (1) passes from the state x_0, $t_0 = 0$ to a predetermined state in a minimum time T^0. Depending on the type of g_T, $f(x, t)$ and the desired final state, the above problem can be formulated differently. Three different formulations (A, B, C) are given, all being variational problems related to the solution of differential equations and possessing certain peculiar features. The solutions which are of interest in practice, are discontinuous. Problem C is called synthesis-problem for optimal systems (s.o.s). In such problems, u^0 should be calculated by means of computers much faster than the duration of the transient process. Below, the problems are solved by the methods developed by the author in earlier works. The linear system

$$\frac{dx}{dt} = P(t)x + Bu + \varphi(t) \qquad (4)$$

is considered. In certain cases of interest, the optimum-control problem reduces to the well-known problem of functional analysis, called "L-problem" in Ref. 12 (N. Akhiyezer, M. Kreyn, O nekotorykh

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voprosakh teorii momentov art. 4, p. 171, GONTI-NVTU, 1935). Thereby, the existence conditions of the a.o.c. can be ascertained as well as its dependence on the system parameters and on x; limiting processes in optimal solutions can be studied which is important for approximate methods of solution. The following two problems are considered in detail:

$$g_r = \max(|u_1(t)|, 0 \leqslant t \leqslant T), \ r = 1, \tag{5}$$

$$g_r = \max\left(\sum_{i=1}^{n} u_i^2(t), 0 \leqslant t \leqslant T\right), \ r = n, \tag{6}$$

whereby problem (6) is auxiliary (to the approximate method of solving problem (5)). By theorems of Ref. 12 (Op.cit.) the following results are obtained: the a.o.c. of (5) is a relay function

$$u_1^0(t) = \operatorname{sign}\left(\sum_{i,j} \lambda_i^0 f_{ij} b_{j1}\right), \tag{8}$$

the a.o.c. of (6) is a continuous vector-function

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$$u_i^0(t) = \left(\sum_{j,k} \lambda_j^0 f_{jk} b_{ki}\right) \Big/ \left(\sum_{i=1}^{n} \left[\sum_{j,k} \lambda_j^0 f_{jk} b_{ki}\right]^2\right)^{\frac{1}{2}}; \tag{9}$$

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the numbers λ^0 and the optimal time T^0 are determined from

$$\gamma^{(1)}(T) = \min\left[\int_0^T \sum_{i}\left|\sum_{j,l} \lambda_j f_{jl} b_{li}\right| dt,\ \sum_{i=1}^{n} c_i \lambda_i = 1\right] = 1, \tag{10}$$

$$\gamma^{(2)}(T) = \min\left[\int_0^T \left[\sum_{i}\left(\sum_{j,l} \lambda_j f_{jl} b_{li}\right)^2\right]^{\frac{1}{2}} dt,\ \sum_{i=1}^{n} c_i \lambda_i = 1\right] = 1 \tag{11}$$

In the case of nonlinear systems, the necessary optimum conditions involve the system with variations along the optimal trajectory:

$$\frac{d\delta x}{dt} = P(t)\delta x + B\delta u, \quad \left((P)_{ij} = \left(\frac{\partial f_i}{\partial x_j}\right)_{x=x(t)}\right). \tag{14}$$

In the linear case, the problem of the existence of an a.o.c. for a given $x = x_0$, reduces to the "L-problem". In the nonlinear case, the existence problem is more complicated. Under the above assumptions one obtains for problem (5) an a.o.c. of relay type. In the linear case, the necessary conditions (as derived above) are, as a rule, also the sufficient optimum-conditions (both locally and in the large). In the nonlinear case however, these conditions are not sufficient. In synthesizing optimal systems, the choice of the constants λ^0 presents a difficult problem. A method of solution to problem (5) is described (with the disturbance $\equiv 0$). Together with system (4), the system

$$\frac{\partial x_i}{\partial t} = \vartheta \sum_{j=1}^{n} p_{ij}(t) x_j + \vartheta b_{i1} u_1 + (1 - \vartheta) \sum_{j=1}^{n} b_{ij} u_j. \qquad (19)$$

is considered. Proceeding from (11), a system of differential equations is set up, viz.

$$\left.\begin{aligned} \frac{dT^0}{d\vartheta} &= F_1(T^0, \{\lambda_i^0\}, \vartheta); \\ \frac{d\lambda_i^0}{d\vartheta} &= F_1(T_0, \{\lambda_i^0\}, \vartheta) \quad (i = 2, \ldots, n). \end{aligned}\right\} \qquad (20)$$

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which satisfies the conditions of existence and uniqueness of solutions. By integration of (20), one obtains

$$T^0 = \lim T^0(\vartheta); \ \lambda_i^0 = \lim \lambda_i^0(\vartheta) \text{ for } \vartheta \to 1,$$

where λ_i^0 determine the a.o.c. (5) according to formula (...). Further, the use of the method of Lyapunov functions in synthesis problems, is considered (for autonomous systems); in particular, problems (5) and (6). Along the optimal trajectory $x(x_0, t, u^0)$, the equation

$$\frac{dT^0(x(x_0, t, u^0))}{dt} = \sum_{j=1}^{n} \frac{\partial T^0}{\partial x_j}(f_j(x) + Bu^0) = \min_{(\alpha,y,u)} \sum_{j=1}^{n} \frac{\partial T^0}{\partial x_j}(f_j(x) + Bu)), \quad (22)$$

holds, i.e. $T^0(x)$ can have the role of the Lyapunov function for the optimal system. Hence for synthesizing optimal systems it is sufficient to know the function $v = T^0(x)$ which satisfies the conditions of Lyapunov's theorem on asymptotic stability. Condition (22) yields a partial differential equation for v. In simple cases

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it is possible to solve the synthesis problem by solving this equation by the method of characteristics. Finally, the use of Lyapunov functions in problem C is considered. A discussion followed; taking part were Ye.A. Barbashin (USSR), R. Kulikovskiy (Poland). There are 12 Soviet-bloc references.

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D206/D304

16,4000 (1031,1121,1344)

AUTHORS: Krasovskiy, N.N., and Lidskiy, Z.A. (Sverdlovsk)

TITLE: Analytical design of controllers in systems with random properties

PERIODICAL: Avtomatika i telemekhanika, v. 22, no. 9, 1961, 1145 - 1150

TEXT: This is a short analytical analysis of a control system undergoing random changes and subjected to random interference, whose bloc diagram is shown in Fig. 1. In it $z(t)$ is the controlled -vectorial quantity, $z_0(t)$ - the required magnitude of this quantity, $x(t) = z(t) - z_0(t)$ - error vector $g(t)$ - activating force $(\xi + \xi)$ - excitation of the regulator, $\gamma(t)$ - interference, $\eta(t)$ - factor determining the random changes in the controlled load A. The quantity ξ is assumed to be known. If between the components x_i of error vector $x(t)$, there exists components not equal to zero

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the stage B produces an additional stabilizing force ξ which governs the transient. The aim of the present article is the analytical determination of quantity ξ. The law of control of $\xi(x, \eta)$ will be determined from the conditions for the minimum of integral evaluation of quality. It is assumed that the equations of the random process in the system written in the coordinates x_i of the error vector x(t), have the form

$$\frac{dx_i}{dt} = \varphi_i[x_1, \ldots, x_n, \eta(t), \xi] + \gamma_i, \quad \xi = \xi[x_1, \ldots x_n, \eta]. \quad (1.1 + 1.2)$$

Function φ_i is assumed to be known and continuous. The function $\eta(t)$ is a random function determining the behavior of stage A at various instant of time t. P[L/Q] is assumed to be the probability of occurrence of L under the condition Q and M[l/q] the mathematical expectation of a random quantity l under the condition Q. Denoting by O(ε) a quantity of a higher small order than quantity ε and by O(ε) - a small quantity of the same ε order, the random

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changes of $\eta(t)$ are described by functions $q(\alpha)$ and $q(\alpha, \beta)$. Functions $q(\alpha)$ and $q(\alpha, \beta)$ satisfy $q(\alpha, -\infty) = 0$, $q(\alpha, \infty) = q(\alpha)$. If $\eta(t)$ can assume only one of the values of $k = (\alpha_1 \ldots \alpha_k)$ then to describe the process it is enough to know the transfer matrix $//p_{ij}//$ where

$$P[\eta(t + \Delta t) = \alpha_j/\eta(t) = c_i] = p_{ij}\Delta t + o(\Delta t),$$
$$q(\alpha_i) = \sum_{j \neq i}^{k} p_{ij}, \quad q(\alpha_i, \beta) = \sum_{j=1}^{m} p_{ij} \quad \text{FOR} \quad \alpha_m \leqslant \beta < \alpha_{m+1}. \qquad (2.3)$$

If the function $q(\alpha, \beta)$ has the probability density $q(\alpha, \beta) = \int_{-\infty}^{\beta} p(\alpha, v)dv$, then

$$P[\beta_1 < \eta(t+\Delta t) < \beta_2, \eta(t+\Delta t) \neq \alpha/\eta(t) = \alpha] = \Delta t \int_{\beta_1}^{\beta_2} p(\alpha, v)\, dv + o(\Delta t). \qquad (2.4)$$

It is further assumed that it is possible to measure $\eta(t)$ and that it is applied to stage B without distortion and delay. Interference γ at the input is assumed to be in the form of random pulses resulting in step changes of the output $\Delta_{\mu} x_i \approx v\mu$, where v_i a random quantity, μ_i - a known function. The mean value of v_i is assumed to be zero, the dispersion $M\left\{v_i^2\right\} = \sigma_i^2 \geqslant 0$ and the correlation coefficients k_{ij} $(M\left\{v_i v_j\right\} = k_{ij}\,\sigma_i\sigma_j)$ are assumed to be known and only the limiting case of the interference, $\lambda \longrightarrow \infty$, $\sigma_i \longrightarrow 0$ at $\lambda\sigma_i^2$ = const is considered. If therefore a certain function $\omega[x_1, \ldots, x_n, \eta, \xi]$ is given which determines the criteria for the transient process

$$I_{\xi}[x_o, \eta_o] = \int_0^{\infty} M\{\omega/x_1 = x_{10}, \ldots, x_n = x_{n0}, \eta = \eta_o \text{ for } t = t_o = 0\}\, dt, \qquad (3.1)$$

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where $\omega = \omega[x_1(t), \ldots, x_n(t), \eta(t), \xi(x(t), \eta(t))]$ the problem is to find the function $\xi = \xi^o(x_1, \ldots x_n, \eta)$ so that the solutions x(t) of system (1.2) and (1.1) satisfy the following conditions: a) The given movement x = 0 has a probability stability; b) The errors (x_{10}) possible in the system, should result in a process, assymptotically stable in its probability correspondingly x = 0; c) The integral Eq. (3.1) for a given control $\xi = \xi^o(x, \eta)$ should have a minimum when compared with its values determined by another ξ. In the 2nd part of the article an approximate method of optimizing the function v is given, in the third part the problem of the minimum r.m.s. error is solved for linear systems. The authors acknowledge the help of A.M. Letov. There are 1 figure, and 32 references: 28 Soviet-bloc and 4 non-Soviet-bloc. [Abstractor's note: 3 of the Russian-language references are translations from English] The 4 references to the English-language publications read as follows: Bellman R. Glicksberg, J. Gross O., Some Aspects of the Mathematical Theory of Controll Processes, Project Rand 1958; R. Bell-

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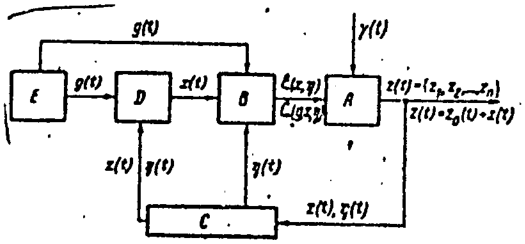
SUBMITTED: March 18, 1961

Fig. 1.



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16.4000 (1103, 1329, 1132)

29242
S/103/61/022/010/001/018
D274/D301

AUTHOR: Krasovskiy, N. N., and Lidskiy, E. A.

TITLE: Analytical design of controllers for random systems II. Optimum-control equations. Approximate method of solution

PERIODICAL: Avtomatika i telemekhanika, v. 22, no. 10, 1961, 1273-1278

TEXT: Optimum-control equations are derived on the basis of the general method of the authors in part I of the article, (Ref. 1: Avtomatika i telemekhanika, v. 22, no. 9, 1961). The concepts and notations are the same as in part I. In Ref. 1 (Op. cit.), rules were formulated which govern the search for the optimum-control law ξ^0 which minimizes the integral performance-criterion

$$I_\xi = \int_0^\infty M\{\omega[x(t), \xi(t)] / x_0, \eta_0, t_0 = 0\} dt = \min_\xi \tag{1.1}$$

of the stochastic control-system

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$$\frac{dx_i}{dt} = \varphi_i \left[x_1, \dots, x_n, \eta (t), \xi \right] \tag{1.2}$$

$$\xi = \xi \left[x_1, \dots, x_n, \eta \right] \tag{1.3}$$

By those rules, ξ^0 is determined from the condition

$$\left[\frac{dM(v^0)}{dt} + \omega\right]_{\xi^0} = \min_\xi \left[\frac{dM(v^0)}{dt} + \omega\right]_\xi = 0, \tag{1.4}$$

where v^0 is a positive-definite, optimum Lyapunov-function. The partial differential equations are derived which are a consequence of of Eq. (1.4). These equations yield

$$\sum_{i=1}^{n} \frac{\partial v(x, \eta)}{\partial x_i} \varphi_i (x, \xi, \eta) + \int_{-\infty}^{\infty} v(x, \beta)\, d_\beta q(\eta, \beta) - \\ - q(\eta)\, v(x, \eta) + \frac{\lambda}{2} \sum_{i,j=1}^{n} \frac{\partial^2 v(x, \eta)}{\partial x_i \partial x_j} k_{ij} \mu_i \mu_j \sigma_i \sigma_j + \omega [x, \xi, \eta] = 0 \tag{1.8}$$

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